

공학수학 에센스

연습문제 풀이 이용 안내

- 본 문제 풀이의 저작권은 마인속과 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

CHAPTER 13 편미분방정식

[13.1 기본 개념]

1. $u = A(y)e^x + B(y)e^{-x}$

2. $u = \int C(x)dx + g(t)$

3. $u = C(y)x + g(y)$

4. $u(x, y) = \frac{1}{y-1}e^{xy} + e^x C(y)$

5. $u(x, y) = A(y)e^{2x} + B(y)xe^{2x} + \frac{1}{9}e^{-x}$

[13.2 열전도방정식]

6. $u(x, t) = \sum_{n=1}^{\infty} \left[\frac{2}{(L-2n)\pi} \sin \frac{(L-2n)\pi}{2} - \frac{2}{(L+2n)\pi} \sin \frac{(L+2n)\pi}{2} \right] \sin \frac{n\pi x}{L} e^{\left(\frac{cn\pi}{L}\right)^2 t}$

7.
$$\therefore u(x, t) = \sum_{n=1}^{\infty} \left[\frac{2L}{n^2\pi^2} \left(\sin \frac{2n\pi}{L} - \sin \frac{8n\pi}{L} + \sin n\pi \right) + \frac{2(L-10)}{n\pi} \cos n\pi \right] \sin \frac{n\pi x}{L} e^{-\left(\frac{cn\pi}{L}\right)^2 t}$$

8.
$$u(x, t) = \sum_{n=1}^{\infty} -\frac{2\alpha}{n\pi} \left(\cos \frac{n\pi}{2} - 1 \right) \sin \frac{n\pi x}{L} e^{-\left(\frac{cn\pi}{L}\right)^2 t}$$

9.
$$\therefore u(x, t) = \sum_{n=1}^{\infty} \left[-\frac{\cos(1+n)\pi}{1+n} + \frac{\cos(1-n)\pi}{1-n} \right] \sin n\pi x e^{-\left(\frac{cn\pi}{L}\right)^2 t}$$

10. $u(x, t) = \frac{\pi^2}{3} - 4\cos x e^{-c^2 t} + \cos 2x e^{-(2c)^2 t} - \frac{4}{9}\cos 3x e^{-(3c)^2 t} + \dots$

11.
$$u(x, t) = 2\cos 2x e^{-\left(\frac{2c}{L}\right)^2 t}$$

12.
$$u(x, t) = \frac{e^{\pi} - 1}{\pi} + \sum_{n=1}^{\infty} \frac{2}{(n^2+1)\pi} \left[(-1)^n e^{\pi} - 1 \right] \cos n\pi x e^{-\left(\frac{cn\pi}{L}\right)^2 t}$$

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$$13. \therefore u(x, t) = \frac{2}{\pi} + \left(-\frac{1}{\pi}\right) \sum_{n=1}^{\infty} \left[\frac{\cos((1+n)\pi)}{1+n} + \frac{\cos(1-n)\pi}{1-n} - \frac{2}{1-n^2} \right] \cos n\pi e^{-(cn)^2 t}$$

[13.3 파동방정식]

$$14. \therefore u(x, t) = \frac{1}{L} \left(L - \frac{1}{4\pi} \sin 4\pi L \right) \cos \frac{2c\pi t}{L} \sin \frac{2\pi x}{L}$$

$$15. u(x, t) = \sum_{n=1}^{\infty} \frac{16}{n^3 \pi^3} \left(1 - (-1)^n \right) \cos \frac{cn\pi t}{2} \sin \frac{n\pi x}{2} = \frac{32}{\pi^3} \left(\cos \frac{c\pi t}{2} \sin \frac{\pi x}{2} + \frac{1}{2\pi} \cos \frac{3c\pi t}{2} \sin \frac{3\pi x}{2} + \dots \right)$$

$$16. \therefore u(x, t) = \sum_{n=1}^{\infty} \left(\frac{(-1)^n \cdot 2}{n\pi} + \frac{6}{n^2 \pi^2} \sin \frac{n\pi}{3} \right) \cos(cn\pi t) \sin(n\pi x)$$

$$17. \therefore u(x, t) = \sum_{n=1}^{\infty} 2 \left[-\frac{4}{3n\pi} \cos \frac{2n\pi}{3} + \frac{3}{n^2 \pi^2} \sin \frac{n\pi}{6} - \frac{3}{n^2 \pi^2} \sin \frac{2n\pi}{3} \right] \cos(cn\pi t) \sin(n\pi x)$$

$$= \left(-\frac{8}{3\pi} \cos \frac{2\pi}{3} + \frac{3}{\pi^2} \sin \frac{\pi}{6} - \frac{3}{\pi^2} \sin \frac{2\pi}{3} \right) \cos(c\pi t) \sin(\pi x)$$

$$+ \left(-\frac{4}{3\pi} \cos \frac{4\pi}{3} + \frac{3}{4\pi^2} \sin \frac{\pi}{3} - \frac{3}{4\pi^2} \sin \frac{4\pi}{3} \right) \cos(2c\pi t) \sin(2\pi x) + \dots$$

$$18. \therefore u(x, t) = \sum_{n=1}^{\infty} \left[\frac{2n}{(n^2+1)\pi} \left(1 - (-1)^n e^{\pi} \right) \cos(cnt) + \frac{1}{cn\pi} \left(-\frac{\cos((1+n)\pi)}{1+n} + \frac{\cos(1-n)\pi}{1-n} - \frac{2n}{1-n^2} \right) \sin(cnt) \right] \sin n\pi x$$

$$19. \therefore u(x, t) = \sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1} \cdot 2}{n} \cos(cnt) + \frac{2}{cn^2 \pi} \left\{ (-1)^{n+1} (1+\pi) + 1 \right\} \sin(cnt) \right] \sin n\pi x$$

$$20. \therefore u(x, t) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left[(-1)^n (\pi - 1) + 1 \right] \cos(cnt) \sin n\pi x$$

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21.

$$\therefore u(x, t) = \sum_{k=1}^{\infty} \frac{8(-1)^{k+1}}{c(2k-1)^3\pi^3} \sin \frac{c(2k-1)\pi t}{2} \sin \frac{(2k-1)\pi x}{2} \\ - \sum_{k=0}^{\infty} \frac{(-1)^k}{c k^2 \pi^2} \sin(c k \pi t) \sin(k \pi x)$$

22. $\therefore u(x, t) = \sum_{n=1}^{\infty} \frac{50}{c n^2 \pi^2} \sin \frac{c n \pi t}{5} \sin \frac{n \pi x}{5}$

[13.4 라플라스 방정식]

23. $\therefore u(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \frac{n \cosh n x + \sinh n x}{n \cosh n \pi + \sinh n \pi} \sin n y$

24. $\therefore u(x, y) = \sum_{n=1}^{\infty} \frac{1}{\sinh \frac{b n \pi}{a}} \frac{4 a^2 n \pi (1-(-1)^n \cos a)}{(a^2 - n^2 \pi^2)^2} \sin \frac{n \pi}{a} x \sin h \frac{n \pi}{a} y$

25.

$$\therefore u(x, y) = \sum_{n=1}^{\infty} \frac{1}{\sinh n \pi} \left[\frac{2}{n \pi} \left((-1)^n - \cos \frac{n \pi}{2} \right) + \frac{8}{n^2 \pi^2} \sin \frac{n \pi}{2} \right] \sin \frac{n \pi}{2} x \\ + \sum_{n=1}^{\infty} \frac{1}{\sinh n \pi} \frac{16}{n^3 \pi^3} \left(1 - (-1)^n \right) \sin \frac{n \pi}{2} y$$

26. $u(x, y) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 32 n}{\pi^2 (1+2n)^2 (1-2n)^2 \sinh(4 n \pi)} \sinh(n \pi y) \sin(n \pi x)$

27. $\therefore u(x, y) = \sum_{n=1}^{\infty} \frac{2[(-1)^{n+1}(n^2+1)-2n]}{\pi^2(1-n)^2(1+n)^2 \sinh(n \pi^2)} \sinh(n \pi y) \sin(n \pi x)$

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28.

$$u(x, y) = \sum_{n=1}^{\infty} (A_n \cosh ny + B_n \sinh ny) \sin nx \\ + \sum_{n=1}^{\infty} (C_n \cosh nx + D_n \sinh nx) \sin ny$$

$$\text{단, } A_n = \frac{4}{n\pi^3} (1 - (-1)^n)$$

$$B_n = \frac{4}{n\pi^3} ((-1)^n - 1) \cosh n\pi / \sinh n\pi$$

$$C_n = \frac{2n((-1)^{n+1} - 1)}{\pi(1+n)(1-n)}$$

$$D_n = -\frac{2n((-1)^{n+1} - 1)}{\pi(1+n)(1-n)} \cosh n\pi / \sinh n\pi$$